Zero Coupon And Implied Rate

PURPOSE

To explain the relationship between Zero Coupon and Implied Forward Rate.

WHY IS THIS IMPORTANT?

Estimating future cash flow arising from an expected interest rate is a component part in the valuation instruments such as fixed rate bonds and interest rate swaps. Such estimates are derived from the zero coupon curve in the form of implied forward rate.

I. DERIVE ZERO COUPON CURVE, GIVEN IMPLIED FORWARD RATES

In this example, we shall assume that the implied forward rate is known. From this, we can compute the equivalent zero coupon rate.

Year	Fwd Rate	FV of $1.00
1	2.4123%	1.02412300
2	2.5567%	1.05030675
3	2.6389%	1.07802330
4	2.7827%	1.10802145
5	2.8231%	1.13930201
6	2.9003%	1.17234518

For this information, $1.00 invested at the start will be have a value at the end of each period as given above.

For example

At the end of Yr 1, the value will be

1 + (1 * r)

= 1 + (1 * 2.4123%)

= 1.02412300

At the end of Yr 2, the value becomes

1.02412300 * (1.02412300 + 2.5566%)

= 1.05030675

etc…

Based on a future value of 1.05030675 at the end of Yr 2, we can compute the expected yield on a two-year zero coupon yield (ZCR_2). This is expressed using the formulae

$$(1 + ZCR_2)^2 = FV_2$$
$$1 + ZCR_2 = FV_2^{\frac{1}{2}}$$
$$ZCR_2 = FV_2^{\frac{1}{2}} - 1$$

In general, the ZCR_n can be computed as follows…

$$ZCR_n = FV_n^{\frac{1}{n}} - 1$$

From this we can compute the ZCR for each Year as follows…

Year	Fwd Rate	FV of $1.00	ZCR
1	2.4123%	1.02412300	2.4123%
2	2.5567%	1.05030675	2.4845%
3	2.6389%	1.07802330	2.5359%
4	2.7827%	1.10802145	2.5976%
5	2.8231%	1.13930201	2.6426%
6	2.9003%	1.17234518	2.6855%

For example

$$ZCR_2 = (1.05030675)^{\frac{1}{2}} - 1 = 2.4845\%$$

An investor will be indifferent between investing at a rate of 2.4845% for 2 years and investing at a rate of 2.4123% and reinvesting the maturing proceeds at 2.5567%, the given implied forward rate, in the second year.

II. DERIVE IMPLIED FORWARD RATES FROM ZERO COUPON RATE

In reality, the Zero Coupon Rate is derived not from the implied forward rate but boot strapped from the yield of Treasury Bill/Bonds and relevant fixed income instruments.

Based on the relationship in I above, we can derive the implied forward rate from a given Zero Coupon Bond using the following formula:

$$r_{t1,t2} = \left(\frac{(1 + r_2)^{d_2}}{(1 + r_1)^{d_1}}\right)^{\frac{1}{d_2 - d_1}} - 1$$

$r_{t1,t2}$ is the forward rate between term t_1 and term t_2,

d_1 is the time length between time 0 and term t_1 (in years)

d_2 is the time length between time 0 and term t_2 (in years)

r_1 is the zero-coupon yield for the time period (0, t_1)

r_2 is the zero-coupon yield for the time period (0, t_2)

Year	Note A	Note B	Note C	Note D
1	1	1.024123000	1.000000000	2.4123%
2	1	1.050306753	1.024123000	2.5567%
3	1	1.078023298	1.050306753	2.6389%
4	1	1.108021452	1.078023298	2.7827%
5	1	1.139302006	1.108021452	2.8231%
6	1	1.172345182	1.139302006	2.9003%

Ref	Remarks
A	$\frac{1}{d_2 - d_1}$ In our example, the periods are 1 year apart.
B	$(1 + r_2)^{d_2}$ This is the numerator item (Current period)
C	$(1 + r_1)^{d_1}$ This the denominator item (Preceding period)
D	$\left(\frac{B}{D}\right)^{A} - 1$ This is the implied forward rate.

Note that the above implied forward rate derived from the above method is the same as those used to derive the Zero Coupon Curve from Section I above.

CHANGE HISTORY

Date	By	Changes
15-Feb-2016	CS	Created.
14-Jun-2016	Richard	Proofread.