Report 8034: Bond YTM and Duration

PURPOSE

To provide the formulae and detail computation used by CS Lucas for computing bond yield to maturity and duration.

WHY IS THIS IMPORTANT?

Allows users to verify the formulae and methodology used by CS Lucas to compute the bond yield and duration.

FORMULA

The formulae used by CS Lucas for price and yield calculation are as follows:

(I) For semi-annual coupon with 6 months or less to maturity:

$$P = \left[\frac{100\left(RV + \frac{CPN}{FREQ}\right)}{100 + \left(\frac{DSM}{E} \times \frac{Y}{FREQ}\right)}\right] - \left(\frac{CPN}{FREQ} \times \frac{DCS}{E}\right)$$

(II) For semi-annual coupon with more than 6 months to maturity:

$$P = \left[\frac{RV}{\left(1 + \frac{Y}{FREQ}\right)^{N-1+\frac{DES}{E}}}\right] + \left[\sum_{k=1}^{N} \frac{\frac{CPN}{FREQ}}{\left(1 + \frac{Y}{FREQ}\right)^{K-1+\frac{DSC}{E}}}\right] - AI$$

Where

AI accrued interest per S$100 face value

DSM days between settlement date and maturity date

DCS days between beginning of current coupon period and settlement date

E number of days in coupon period where settlement occurs

DSC E – DCS = days from settlement date to next 6 month coupon date

N number of coupons payable between settlement date and maturity date

CPN annual coupon rate (as a percentage)

Y annual yield (as a percentage)

P price per S$100 face value

RV redemption value

FREQ The number of coupon payments per year.

FREQ = 1 for annual coupon payments

FREQ = 2 for semi-annual coupon payments

FREQ = 4 for quarterly coupon payments

Source:

http://www.sgs.gov.sg/~/media/SGS/SGSRulesMktPractices.pdf

https://support.office.com/en-us/article/PRICE-function-3ea9deac-8dfa-436f-a7c8-17ea02c21b0a

BOND REFERENCE

The bond used to illustrate the computation in this guide is an actual issue by the Singapore Government. Details are as follows:

Reference Code	NA12100N
ISIN	SG3254976487
Issuer	Singapore Government
Issue Date	02-Apr-12
Maturity Date	01-Apr-42
Currency	SGD
Annual CouponFrequency	Semi Annually
Coupon Type	Fixed

Annual Coupon Rate (% p.a.)	2.75

QUERY

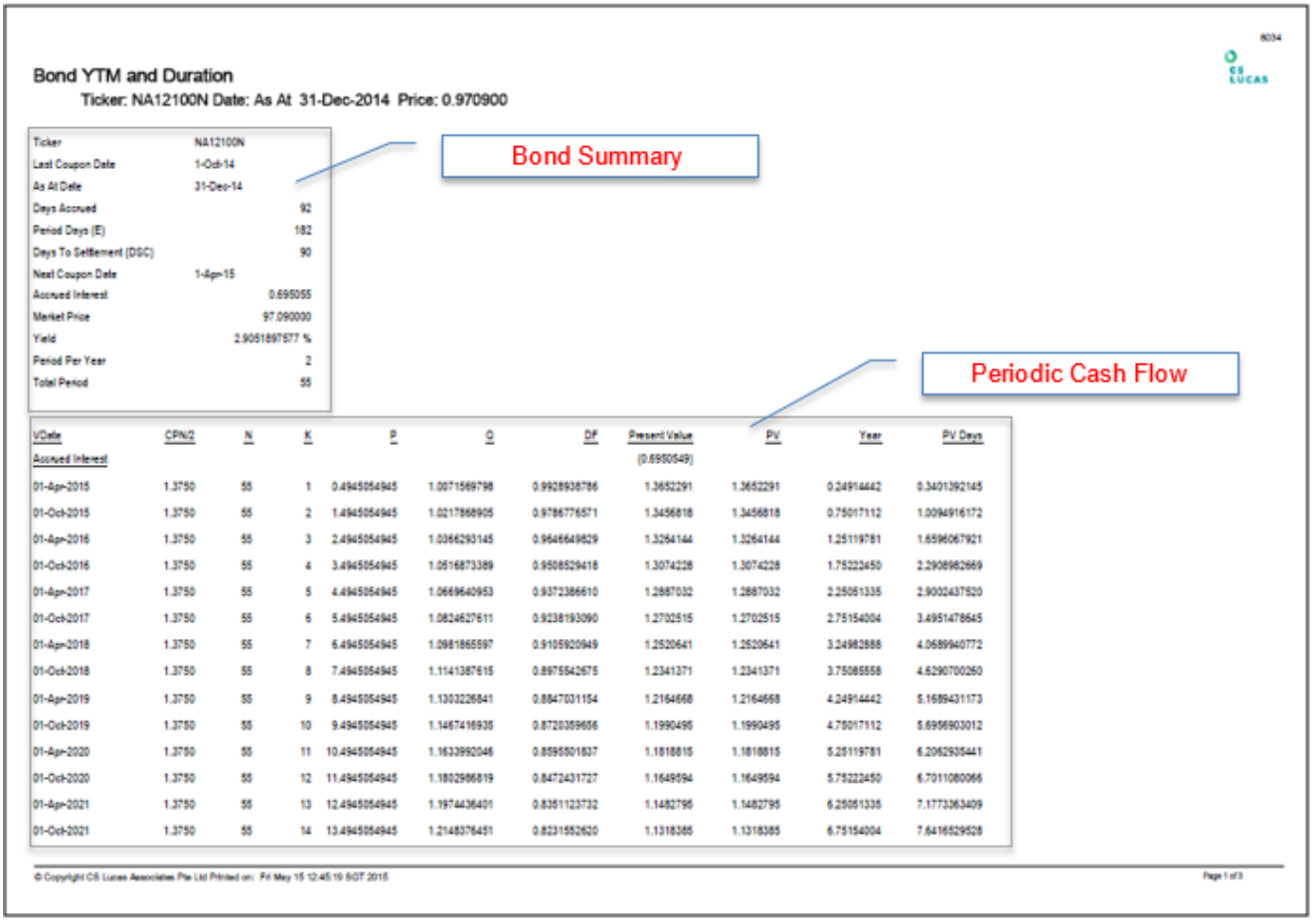
1. Navigate to Report 8034: Bond YTM and Duration.

2. Enter the Ticker of bond and required As At Date for the computation.

3. You may enter a price for the purpose of the computation. If you would like the system to use the latest archived priced to As At Date, leave the price field blank.

4. Click on the required format.

5.The report has two sections: Bond Summary and Periodic Cash Flow.



BOND SUMMARY

Ticker	NA12100N
Last Coupon Date	1-Oct-14
As At Date	31-Dec-14
Days Accrued	92

Period Days (E)	182
Days To Settlement (DSC)	90
Next Coupon Date	1-Apr-15
Accrued Interest	0.695055
Market Price	97.09
Yield	2.9051897577%
Period Per Year	2
Total Period	55

Last Coupon Date The date when before the As At Date of the query when the coupon was paid.

Days accrued Number of days from last coupon date up to and including the As At Date.

1-Oct-14 to 31-Dec-14 + 1 day.

Period Days Number of days in the current coupon period (1-Oct-14 to 1-Apr-15)

Days to Settlement Number of days to the next coupon date (1-April-15)

Accrued Interest

$$\frac{Coupon}{Period\ Per\ Year} \times \frac{Days\ Accrued}{Period\ Days} = \frac{2.75\%}{2} \times \frac{92}{182} = 0.695055 \text{ (full precision not shown)}$$

Accrued interest is deducted from the present value of future cash flow on the bond to determine the bond market value. See Periodic Cash Flow below.

Yield See explanation below on the derivation.Yield See explanation below on the derivation.

PERIODIC CASH FLOW

VDate	CPN/2	N	K	P	Q	DF	Present Value	PV	Year	PV Days
Accrued Interest							(0.6950549)			
01-Apr-2015	1.3750	55	1	0.4945054945	1.0071569798	0.9928938786	1.3652291	1.3652291	0.24914442	0.3401392145
01-Oct-2015	1.3750	55	2	1.4945054945	1.0217868905	0.9786776571	1.3456818	1.3456818	0.75017112	1.0094916172
01-Apr-2016	1.3750	55	3	2.4945054945	1.0366293145	0.9646649829	1.3264144	1.3264144	1.25119781	1.6596067921
01-Oct-2016	1.3750	55	4	3.4945054945	1.0516873389	0.9508529418	1.3074228	1.3074228	1.75222450	2.2908982669
01-Apr-2017	1.3750	55	5	4.4945054945	1.0669640953	0.9372386610	1.2887032	1.2887032	2.25051335	2.9002437520
01-Oct-2017	1.3750	55	6	5.4945054945	1.0824627611	0.9238193090	1.2702515	1.2702515	2.75154004	3.4951478645
01-Apr-2018	1.3750	55	7	6.4945054945	1.0981865597	0.9105920949	1.2520641	1.2520641	3.24982888	4.0689940772
01-Oct-2018	1.3750	55	8	7.4945054945	1.1141387615	0.8975542675	1.2341371	1.2341371	3.75085558	4.6290700260

....rows omitted...

02-Apr-2040	1.3750	55	51	50.4945054945	2.0713579018	0.4827750912	0.6638158	0.6638158	25.25393566	16.7639615036
01-Oct-2040	1.3750	55	52	51.4945054945	2.1014463406	0.4758627335	0.6543113	0.6543113	25.75222450	16.8499714930
01-Apr-2041	1.3750	55	53	52.4945054945	2.1319718425	0.4690493467	0.6449429	0.6449429	26.25051335	16.9300822045
01-Oct-2041	1.3750	55	54	53.4945054945	2.1629407563	0.4623335138	0.6357086	0.6357086	26.75154004	17.0061840674
01-Apr-2042	101.3750	55	55	54.4945054945	2.1943595230	0.4557138379	46.1979903	46.1979903	27.24982888	1,258.8873304747
							97.0900477	97.7851026		1,865.9324519071

Each column in the Periodic Cash Flow section is an intermediate working in Formula II above.

Below are the detailed computations using the coupon cash flow on 1-Oct-2015 (Highlighted).

Column	Explanation
CPN/2	$\frac{CPN}{FREQ} = \frac{2.75}{2} = 1.375$
N	The total number of coupon periods outstanding after the As at Date to Maturity Date.
K	The cardinal index of the coupon items
P	$K - 1 + \frac{DSC}{E} = 2 - 1 + \frac{92}{182} = 1.495054945$
Q	$\left(1 + \frac{Y}{FREQ}\right)^{P} = \left(1 + \frac{2.9051897577\%}{2}\right)^{1.495054945} = 1.0217868905$
DF	$\frac{1}{Q} = \frac{1}{1.0217868905} = 0.9786776571$
Present Value	Future coupon $\frac{CPN}{2} \times DF = 1.375 \times 0.9786776571 = 1.3456818$ On maturity $RV + \frac{CPN}{2} \times DF = 101.375 \times 0.4557138379 = 46.1979903$

YIELD

Using Formula II above, the price of a bond is the sum of the Present Value column in the Periodic Cash Flow less the Accrued Interest. This total is

dependent on the value of Y used in the determination of Column Q.

The Yield of the bond is the value for Y such that the sum of the Present Value column equals to the market price of the bond.

In the illustration above, for a bond value of 97.07 (with minor rounding), Y has to be 2.9051897577%.

Therefore, the yield of the bond is 2.9051897577%.

MACAULAY DURATION

III) Macaulay Duration is given by the formula

$$Macaulay\ Duration = \sum_{i=1}^{n} t_i \frac{PV_i}{V}$$

Where

PV_t Present value of bond cash flow (coupon and/or redemption) at period t.

I Time to each cash flow in years.

k Periods to maturity.

V The present value of all future cash payments from the bond.

Source:

https://en.wikipedia.org/wiki/Bond_duration

Below are detailed computations using cash flow on 1-Oct-2015 (Highlighted).

Column	Explanation
Present Value	Future coupon $\frac{CPN}{2} \times DF = 1.375 \times 0.9786776571 = 1.3456818$ On maturity $RV + \frac{CPN}{2} \times DF = 101.375 \times 0.4557138379 = 46.1979903$
Year	$\frac{CF\ date - As\ at\ date}{365.25} = \frac{31\ Dec\ 14\ to\ 1\ Oct\ 15}{365.25} = \frac{274}{365.25} = 0.75017112$
PV Days	$PV \times Year = 1.3456818 \times 0.75017112 = 1.0094916172$

$$Macaulay\ Duration = \sum_{t=1}^{\kappa} \frac{1{,}865.9324519071}{97.7851026} = 19.082$$

MODIFIED DURATION

IV) Modified Duration is given by the formula

$$Modified\ Duration = \frac{Macaulay\ Duration}{1 + \frac{Y}{FREQ}}$$

$$= \frac{19.082}{1 + \frac{2.90518975 77\%}{2}} = 18.8088$$

FREQUENTLY ASKED QUESTIONS

FAQ01. How does the computation of the YTM in this guide compare with using Excel?

The YIELD function in Excel is used to determine the yield of a bond.

Information on the use of this function can be found at: https://support.office.com/en-us/article/YIELD-function-f5f5ca43-c4bd-434f-8bd2-ed3c9727a4fe

A comparison between the Yield Function and the CS Lucas methodology is as follows:

Component	Inputs
settlement	31-Dec-14
maturity	01-Apr-42
rate	2.750%
pr	97.090000
redemption	100
frequency	2
basis	1
Yield-to-Maturity	2.9051818%
Per CS Lucas	2.9051898%
Absolute Difference	0.000008%
Percentage Difference	0.000273%

FAQ02. How does the computation of the Macaulay Duration in this guide compare with using Excel?

The DURATION function in Excel is used to determine the Macaulay duration of a bond.

Information on the use of this function can be found at: https://support.office.com/en-us/article/DURATION-function-b254ea57-eadc-4602-a86a-c8e369334038

Comparison between the DURATION Function and the duration using CS Lucas methodology is as follows:

Component	Inputs
settlement	31-Dec-14
maturity	01-Apr-42
rate	2.750%
yield	2.9051818%
frequency	2
basis	1
Macaulay Duration	19.0813
Per CS Lucas	19.0820
Absolute Difference	0.074687%
Percentage Difference	0.003914%

RELATED INFORMATION

General Formatting For All Reports

CHANGE HISTORY

Date	By	Changes
15-Feb-2016	CS	Created.
16-Jun-2016	Douglas	Proofread.